JUNE 2008 C2

1) $f(x) = 2x^3 - 3x^2 - 39x + 20$

if $(x+4)$ is a factor $\therefore f(-4) = 0$

$f(-4) = 2(-4)^3 - 3(-4)^2 - 39(-4) + 20$

$= -128 - 48 + 156 + 20 = 0$

$\therefore (x+4)$ is a factor

b)

$$
\begin{array}{r|l}
 & 2x^2 - 11x + 5 \\
\hline
x+4 & 2x^3 - 3x^2 - 39x + 20 \\
 & 2x^3 + 8x^2 \\
 & \quad -11x^2 - 39x \\
 & \quad -11x^2 - 44x \\
 & \qquad 5x + 20 \\
 & \qquad 5x + 20 \\
 & \qquad\quad 0
\end{array}
$$

$(x+4)(2x^2 - 11x + 5)$

$x^2 - 11x + 10$

$(x-10)(x-1)$

$(x-5)(2x-1)$

$\boxed{(x+4)(x-5)(2x-1)}$

2 a) $y = \sqrt{5^x + 2}$

x	0	0.5	1	1.5	2
y	1.732	2.058	2.646	3.630	5.196

b) $\frac{1}{2} \times 0.5 \left[(1.732 + \overset{5.196}{\cancel{2.058}}) + 2(2.058 + 2.646 + 3.630) \right]$

$= \underline{5.899}$

3a) $(1+ax)^{10}$

$= 1 + 10ax + \frac{10\times9}{2}(ax)^2 + \frac{10\times9\times8}{3!}(ax)^3$

$= 1 + 10ax + 45a^2x^2 + 120a^3x^3$

b) coeff of $x^3 = x^2 \times 2$

$120a^3 = 2\times45a^2$

$120a = 90$

$a = \frac{3}{4}$

4a) $5^x = 7$

$\log_5 7 = x \qquad x = \underline{1.21} \quad (3\ s.f)$

b) let $5^x = m$

$m^2 - 12m + 35 = 0$

$(m-7)(m-5)$

$m = 7$ or $m = 5$

$5^x = 7$ or $5^x = 5$

$\underline{x = 1.21}$ or $\underline{x = 1}$

5a) $r = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2} = \sqrt{(3-1)^2 + (8-3)^2}$

$= \sqrt{2^2 + 5^2}$

$= \sqrt{29} \quad \therefore r^2 = 29$

$\boxed{(x-3)^2 + (y-1)^2 = 29}$

b) line CP gradient : $\frac{3-1}{8-3} = \frac{2}{5}$

m of perpendicular $= -\frac{5}{2}$

$$y - y_1 = -\frac{5}{2}(x - x_1)$$

$$y - 3 = -\frac{5}{2}(x - 8)$$

$$2y - 6 = -5x + 40$$

$$\boxed{2y = -5x + 46}$$

6a) $a = 5$ $r = \frac{4}{5}$

$u_{20} = 5 \times \left(\frac{4}{5}\right)^{19} = 0.072$ (3 dp)

b) $S_\infty = \frac{a}{1-r} = \frac{5}{1-\frac{4}{5}} = \underline{25}$

c) $S_k > 24.95$

$$\frac{a(1-r^k)}{1-r} > 24.95$$

$$\frac{5(1-0.8^k)}{0.2} > 24.95$$

$$1 - 0.8^k > 0.998$$

$$-0.8^k > -0.002$$

$$0.8^k > 0.002$$

$$\log 0.8^k > \log 0.002$$

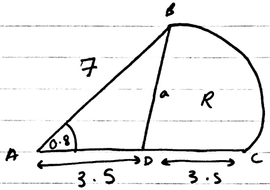
$$k \log 0.8 > \log 0.002$$

$$k > \frac{\log 0.002}{\log 0.8}$$

d) $k > 27.85...$ $\therefore$ $k = 28$

7a) arc length $= r\theta = 7 \times 0.8 = \underline{\underline{5.6}}$ cm

b) area sector $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 7^2 \times 0.8 = \underline{19.6 \text{ cm}^2}$



c)
length $\overrightarrow{BD}$: cosine rule

$$a^2 = 7^2 + 3.5^2 - 2 \times 7 \times 3.5 \times \cos(0.8)$$
$$a^2 = 27.11...$$
$$a = 5.206858...$$

perimetere $= a + \text{arclength} + 3.5$
$= 5.2068... + 5.6 + 3.5 = \underline{\underline{14.3 \text{cm}}}$ (3.s.f)

d) area R = area of sector - area of triangle

$= 19.6 - \frac{1}{2} \times 7 \times 3.5 \times \sin 0.8$

$= 19.6 - 8.7876...$

$= \underline{\underline{10.8 \text{ cm}^2}}$ (3.s.f)

8 a) A = turning point $\therefore \frac{dy}{dx} = 0$

$$y = 10 + 8x + x^2 - x^3$$

$$\frac{dy}{dx} = 8 + 2x - 3x^2$$

$$8 + 2x - 3x^2 = 0 \qquad \times -1$$

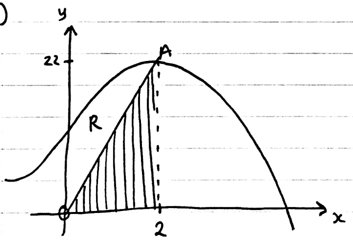
$$3x^2 - 2x - 8 = 0$$

$$(x-2)(3x+4)$$

$$x = 2 \quad \text{or} \quad x = -\frac{4}{3}$$

x is positive $\therefore x = 2$

b)



when $x = 2$

$y = 10 + 8(2) + 2^2 - 2^3$

y coord of $A = 22$

$$\text{area } R = \int_0^2 - \text{triangle}$$

$$= \int_0^2 10 + 8x + x^2 - x^3 \, dx - \frac{1}{2} \times 2 \times 22$$

$$= \left[10x + 4x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4\right]_0^2 - 22$$

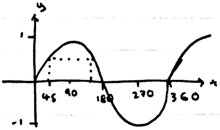
$$= \frac{104}{3} - 22$$

$$\boxed{= \frac{38}{3} \text{ units}^2}$$

9a) $0 \leq x < 360$

$\sin(x-20) = \frac{1}{\sqrt{2}}$

$x-20 = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45$



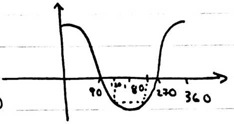
$x-20 = 45, (180-45)$
$= 45, 135$

$\boxed{x = 65, 155}$

b) $\cos 3x = -\frac{1}{2}$

$3x = \cos^{-1}(-\frac{1}{2})$

$3x = 120$



$3x = 120, 240, 480, 600$
$840, 960,$

$120 - 90 = 30$
$270 - 30 = 240$

$\boxed{x = 40, 80, 160, 200, 280, 320}$

* root is repeated every 360°